

On bubbles of nothing in AdS/CFT

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ABSTRACT: We discuss non-perturbative instabilities, mediated by bubbles of nothing, in the context of the AdS/CFT correspondence. By exploring the phase diagram of such decays we give an interpretation of the process in terms of an effective potential in a quantum mechanics of a single variable, a winding mode condensate. The decay corresponds to a conventional first order transition, the true vacuum being a non-geometrical "nothing" state.

KEYWORDS: D-branes, Black Holes in String Theory.

JHEP09(2007)089

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1. Introduction and motivation

The AdS/CFT correspondence (for a review see [1]) provides a non-perturbative background independent definition of quantum gravity in asymptotically AdS spaces. It is useful then to investigate quantum gravity issues in this context. In this paper we examine non-perturbative instabilities mediated by bubbles of nothing [2, 3]. Semiclassical analysis suggests these correspond to vacuum decay, and indeed the false vacuum was identified in [4] as the topological black hole [5]. It is interesting then to discuss the process, and its place in the full non-perturbative definition of the theory. One goal is to clarify the role of the semi-classical analysis in the full theory, as such non-perturbative instabilities may play a crucial role in flux compactifications or in the context of eternal inflation.

In our context, the dual field theory is a conventional (supersymmetric, conformal) field theory formulated in curved space, namely $dS_3 \times S^1$ [4]. This suggests, as already mentioned in [4], that long distance features of the theory are captured in the matrix quantum mechanics of the lowest lying mode, which is the Wilson line wrapping the circle, which we call U . The time in that quantum mechanics is the non-compact time direction of the deSitter space.¹

The spacetimes discussed are not supersymmetric they involve typically various other instabilities, such as tadpoles for massless fields,² and localization on the sphere [4] for the $AdS_5 \times S^5$ case. Thus, a refinement of the above statement is needed: while the complete description of the system, including the aforementioned effects, requires more degrees of

¹In particular we do not expect the features of the decay to be captured by matrix integrals, such as the ones in [7], appropriately analytically continued, as suggested for example in [4, 10]. The process of bubble nucleation is time dependent, therefore degrees of freedom along the time direction of dS_3 should not be integrated out. More on that in [6].

²We thanks Sunny Itzhaki for conversations on this topic.

freedom, we suggest (following [4]) that the process of bubble of nothing nucleation is captured by concentrating on the winding mode condensate u , integrating out all other degrees of freedom.

Our purpose in this paper is then to collect evidence that the reduction to a quantum mechanics of a single degree of freedom u (defined as the condensate of the $Tr(U)$) captures the essential features of the process in the supergravity (large 'tHooft coupling) limit.³ To this end we examine the phase diagram of the theory as function of the circle's radius and three R-charge chemical potentials.⁴ We find that generically the bulk analysis indicates a first order transition. The transition becomes enhanced (the bounce action becomes small) at specific loci in the phase diagram; we find that in precisely those loci the bulk developed a winding tachyon, which is the bulk mode dual to u . These loci then correspond to the disappearance of the barrier in the effective potential, causing the instability to become perturbative and driven by the process of (winding) tachyon condensation. This is taken to be evidence to the conjecture [4] that the non-perturbative instability can be captured in the quantum mechanical model of the winding condensate u .

This analysis can be taken as an evidence for the conjecture in [8, 9] that the end point of the tachyon condensation is indeed the bubble of nothing. Inside that bubble is the true vacuum, the "nothing state", which is then a winding mode condensate. That state is inherently stringy, and does not possess a conventional spacetime interpretation. This is despite the fact that no strong curvatures exist in the bubble spacetime. It would be interesting to further discuss this non-geometrical phase in terms of its field theory dual, understanding for example the disappearance of spacetime in this state.

The outline of this note is as follows. In the next section we discuss the general bubble with up to three chemical potentials turned on. We demonstrate the features of the phase diagram for a few specific cases, and discuss the general qualitative features. Finally, we discuss the explanation of all these features in terms of the quantum mechanics of the variable u and its purported effective potential. We find that all those qualitative features, such as the phase boundaries, can be explained by a conventional quantum mechanical model, where the process is simply the familiar decay of the false vacuum. We present our conclusions and directions for future research in the final section.

2. Bubbles of nothing in AdS/CFT

In this section we review and extend the bulk analysis of the decays mediated by the various bubble of nothing (BON) solutions. We focus on the qualitative features of the relevant gauge theory - in all cases the maximally supersymmetric Yang-Mills theory, formulated in curved space, in the large N and strong 'tHooft coupling limits. Those qualitative features will be compared to those in the weakly coupled gauge theory in [6].

We will concentrate on exploring the phase diagram, looking for the regions in parameter space where the instability exists, and interpret these qualitative features in terms

³In a companion paper [6] we will calculate the effective action of that matrix quantum mechanics in the weak 'tHooft coupling, small volume limit, following the work in [7].

⁴For previous analysis of the phase diagram see [11], for discussion of charged bubbles see also [13].

of a candidate effective potential, such as the one existing in the dual gauge theory. The non-perturbative instability entails an existence of an appropriate bounce solution, a Euclidean solution of the equations of motion with a single (non-conformal) negative mode. The negative mode signals the existence of a local maximum in the effective potential. Additionally we compare the Euclidean action of the bounce with that of the false vacuum: in order for the decay rate is small, corresponding to a meta-stable false vacuum, the action of the bounce has to be higher than that of the false vacuum. When the actions become comparable the false vacuum is no longer meta-stable.

2.1 R-charged bubbles

Let us start with the most general black hole metric carrying up to 3 unequal R-charges. The bubble solutions we are interested in are formed by double analytic continuation as described below. The Lorentzian black hole metric is [12]

$$ds^2 = -(H_1 H_2 H_3)^{-2/3} f dt^2 + (H_1 H_2 H_3)^{1/3} (f^{-1} dr^2 + r^2 d\Omega_{3,k}), \tag{2.1}$$

where

$$f = k - \frac{\mu}{r^2} + \frac{r^2}{l^2} H_1 H_2 H_3, \quad H_i = 1 + \frac{q_i}{r^2}, \quad i = (1, 2, 3), \tag{2.2}$$

Here q_i are 3 charge parameters, related to the physical charges in a manner specified below. To obtain a bubble of nothing solution we perform double analytic continuation. The reality conditions on all fields distinguish between the bubble interpretation and the more familiar black hole (thermal) one, which was analyzed in [12], thus resulting in a different phase diagram. In our case the circle parameterized by $\chi = it$ is interpreted as a spatial direction of the geometry, rather than Euclidean time, therefore the gauge field component in that direction A_χ has to be real. To ensure that we take the charge parameters q_i to be negative, as opposed to having them take positive values for the black holes analyzed in [12].

The Euclidean metric approaches asymptotically $M_k \times S^1$, where S^1 is a circle of asymptotic radius β (which is a parameter of the solution), and M_k is an homogeneous space of constant curvature k and metric $d\Omega_{3,k}$. For the case $k = 1$ it is S^3 , if $k = 0$ it is R^3 , and if $k = -1$ it is a hyperbolic space H_3 . Upon analytic continuation, one of the coordinate of M_k becomes timelike, and the solution resemble a bubble, with the S^1 being interpreted as a spatial circle. In the following we shall concentrate on the solutions with $k = 1$, in which case the analytically continued metric on M_k is that of dS_3 , the 3-dimensional deSitter space.

In addition to the metric the solution has three scalar fields X^i and gauge fields A_μ^i , which are of the form

$$X^i = H_i^{-1} (H_1 H_2 H_3)^{1/3}, \quad A_t^i = \frac{\tilde{q}_i}{r^2 + q_i} + \phi_i, \quad i = 1, 2, 3. \tag{2.3}$$

The constants ϕ_i are adjusted such that the gauge potential at the Euclidean origin is zero. In that case they equal the gauge potentials at infinity, and thus are parameters of the dual gauge theory.⁵

⁵No such parameters, or charges, are associated with the scalar fields X_i .

The physical charges \tilde{q}_i are given in terms of the parameters q_i as

$$\tilde{q}_i^2 = q_i(r_+^2 + q_i) \left[1 + \frac{1}{r_+^2} \prod_{j \neq i} (r_+^2 + q_j) \right], \quad (2.4)$$

where r_+ is the location of the outer horizon, namely the largest root of $f(r)$ defined above. We will mostly work in "grand-canonical ensemble" where the fixed quantities are the potentials at infinity, given by

$$\phi_i \equiv -A_t^i(r_+) = -\frac{\tilde{q}_i}{r_+^2 + q_i}. \quad (2.5)$$

Using the notations of [12] we take ϕ_i to be purely imaginary, which ensures the correct reality conditions upon analytic continuation to the Lorentzian bubble spacetime.

To identify the false vacuum, decaying via the Euclidean solution, interpreted as a bounce, we look at the asymptotic behavior of the fields. The metric behaves asymptotically the same as in the solutions in [4], therefore in all cases the spacetime decaying is the topological black hole which has a non-contractible circle in the geometry (it is obtained from AdS by appropriate identifications, as discussed in [4]). The scalars fall off rapidly at infinity, and the gauge fields approach a constant. We conclude therefore that the false vacuum is the topological black hole with constant gauge potentials around the non-contractible circle.⁶

To map out the general phase diagram we are first interested in the region of parameter space for which the Euclidean solution exists and has a single non-conformal negative mode. In all cases we fix the asymptotic radius β and the value of the potentials at infinity ϕ_i .

In our coordinates there is a possible conical singularity at $r = r_+$. Demanding regularity at the Euclidean origin determines as usual the asymptotic radius β . In this case we find

$$\beta = 2\pi \frac{r_+^2 \sqrt{\prod_i (r_+^2 + q_i)}}{2r_+^6 + (1 + \sum_i q_i)r_+^4 - \prod_i q_i}, \quad (2.6)$$

where we set for convenience the AdS radius to unity, $l = 1$. Together with the formulas (2.4), (2.5) this determines the region of parameter space (spanned by β and ϕ_i for which a candidate Euclidean solution exists).

To confirm that a candidate Euclidean solution is indeed a bounce one needs to check the existence of a non-conformal negative mode. In general this is a difficult problem, solved for the uncharged case in the classic reference [14]. However, it is known that "thermodynamic instability" means that the solution is a local maximum of the free energy, and therefore it is a sufficient condition for the existence of a negative mode [15]. Therefore, in order to check for the possibility of a negative mode we check the Hessian of the Euclidean action (with respect to the field theory parameters) in the region of parameter space relevant for the bubble interpretation.

⁶Furthermore the variables $|\phi_i|$ are periodic in units of β^{-1} .

Finally, to check for the existence of a barrier in field space we calculate the Euclidean action (relative to the false vacuum which is locally simply AdS space). The Euclidean action then can be calculated directly from the above solution, it is found to be [12]

$$I = \beta(M - \sum_{i=1}^3 \tilde{q}_i \phi_i) - S, \quad (2.7)$$

where the mass M is given by

$$M = \frac{3}{2}\mu + \sum q_i = \frac{3}{2} \left(r_+^2 + r_+^4 \prod_{i=1}^3 H_i \right) + \sum_{i=1}^3 q_i, \quad (2.8)$$

and the "entropy"⁷ S is

$$S = \frac{A}{4G_N} = 2\pi \sqrt{\prod (r_+^2 + q_i)}. \quad (2.9)$$

2.2 Uncharged case

The simplest case of the uncharged bubble was discussed extensively in [3, 4], and we discuss it here briefly. In that case the Euclidean metric is

$$ds^2 = f d\chi^2 + f^{-1} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\Omega_2), \quad (2.10)$$

where

$$f = 1 - \frac{\mu}{r^2} + r^2. \quad (2.11)$$

The phase diagram is one dimensional, characterized by the size of the asymptotic circle β , which is given by

$$\beta = \frac{2\pi r_+}{1 + 2r_+^2} \quad (2.12)$$

where the horizon location r_+ can take any positive value. We see then that since this function attains a maximum at $\beta_{\text{crit}} = \pi/\sqrt{2}$, there cannot be a non-perturbative instability for $\beta > \beta_{\text{crit}}$. For $\beta < \beta_{\text{crit}}$ one has two possible solutions (two values of r_+), and one has to check for the existence of negative mode.

The Euclidean action as a function of the parameter r_+ is given by [3]

$$I_{\text{Euc}} = -\frac{\pi r_+^3 (r_+^2 - 1)}{1 + 2r_+^2}, \quad (2.13)$$

where we have subtracted off the Euclidean action of the topological black hole (the false vacuum). As explained above, to check for the existence of a negative mode we calculate the Hessian of the action, in this case this is simply the second derivative at fixed β ⁸

$$\frac{\partial^2 I_{\text{Euc}}}{\partial (r_+^2)^2} \Big|_{\beta} = \frac{3\pi(2r_+^2 - 1)}{2r_+(1 + 2r_+^2)} \quad (2.14)$$

⁷This quantity has no interpretation as entropy in the analytic continuation leading to the bubble solution.

⁸This quantity would be the fixed temperature specific heat in the thermal interpretation.

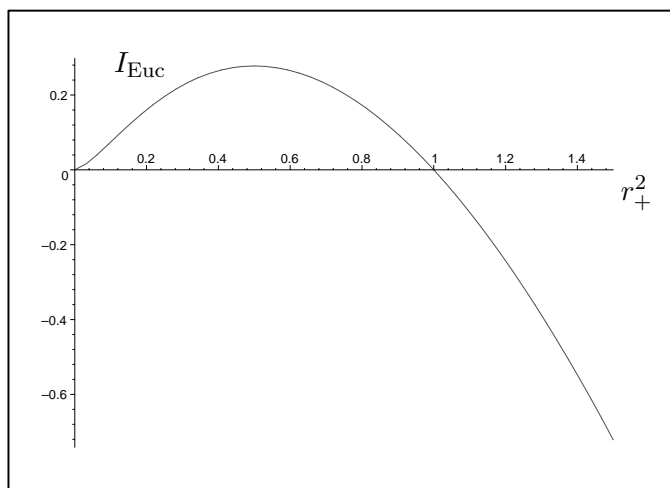


Figure 1: The Euclidean action of the bubbles of size r_+ . When the action is positive the false vacuum is metastable. The conditions for negative Euclidean mode are satisfied for $r_+^2 < \frac{1}{2}$.

We see that for $\beta < \beta_{\text{crit}}$ there are two possible solutions, and one of them has a negative mode, corresponding to an instability. Therefore we conclude that for every $\beta < \beta_{\text{crit}}$ a bounce exists.

Additionally, as showed in figure 1, the Euclidean bubble has positive action (relative to the false vacuum, that is the topological black hole) for $r_+ < 1$, which includes all the relevant region of instability. The action becomes small, corresponding to the disappearance of the barrier, for small radii β , signalling the onset of tachyon condensation as an alternative mode of instability, as discussed below.

To make connection with the conjecture in [8], let us discuss the range of parameters for which a winding tachyon appears in the geometry. Since the analysis depends only on the geometry, and not on the background fields, it applies to all cases below as well.

We start with the false vacuum, namely the topological black hole [4]

$$ds^2 = \left(1 + \frac{r^2}{l^2}\right)^{-1} dr^2 + \beta^2 \left(1 + \frac{r^2}{l^2}\right) d\chi^2 + r^2 d\Omega_3, \tag{2.15}$$

where we have restored the AdS radius l , and β is the periodicity of the circle, so that $\chi \sim \chi + 2\pi$. The size of the circle is

$$L(r) = \beta \sqrt{1 + \frac{r^2}{l^2}}. \tag{2.16}$$

It goes to a constant at $r \rightarrow 0$, and to infinity at the boundary.

For a winding tachyon to condense, or at least for us to be able to use the flat space analysis, two conditions are needed. First, the size of the circle has to reach the string scale l_s , which happens here for small enough values of r . Secondly the variation of the circle's radius ($\frac{L'}{L}$) at that point defines another mass scale which has to be smaller than

all other mass scales for the flat space analysis to be applicable. In particular it has to be smaller than the curvature scale of AdS, l^{-1} , for the analysis to be valid.

These two conditions (for $l \gg l_s$) are satisfied in the regime $\beta < l_s$. In the units employed above ($l = 1$) this translates into $\beta < \lambda^{-\frac{1}{4}}$, which is a very narrow region in the phase diagram at large λ . Consequently the non-perturbative instability exists for a wide range of radii, spanning the range roughly between the string scale and the AdS radius (which are widely separated for large λ).

We note that the region where the winding mode becomes tachyonic coincides with the locus of the phase diagram for which the Euclidean action becomes small signalling the disappearance of the barrier in field space⁹(up to unknown factors of order one). This is in support of the conjecture that the non-perturbative instability is driven by the winding mode, as suggested already in [4].

2.3 One charge case

Now let us turn on a single charge, say $q_1 = q$, and $q_2 = q_3 = 0$, the field theory parameters are given by

$$\begin{aligned}\tilde{q}^2 &= q(r_+^2 + q)(1 + r_+^2), \\ \phi &= -\frac{\tilde{q}}{r_+^2 + q}, \\ \beta &= \frac{2\pi\sqrt{r_+^2 + q}}{2r_+^2 + 1 + q}\end{aligned}\tag{2.17}$$

The bubble solution, obtained by double analytic continuation from the black hole $t \rightarrow i\chi$, $\theta \rightarrow \frac{\pi}{2} + i\tau$, is given by [11]

$$ds^2 = H^{-2/3} f d\chi^2 + H^{1/3} (f^{-1} dr^2 - r^2 d\tau^2 + r^2 \cosh^2 \tau d\Omega_2),\tag{2.18}$$

where

$$H = 1 + q/r^2, \quad f = 1 - \frac{\mu}{r^2} + r^2 H.\tag{2.19}$$

and $q < 0$ in the bubble interpretation. And it is easy to get the action and Hessian

$$\begin{aligned}I &= \beta(M - \tilde{q}\phi) - S = -\frac{\pi R\sqrt{R+q}(R-1+q)}{2R+1+q}, \\ \text{Hessian} &= \frac{\pi^2(3R^2 + 3(1+q)R + 4q)(2R^2 + (1+q)R - (1-q)^2)}{4q(2R+1+q)^2(R+q)(1+R)}.\end{aligned}\tag{2.20}$$

where we defined $R = r_+^2$.

First we look at the existence of bubbles in the parameter space spanned by β and $\beta|\phi|$. This is the shaded region in figure 2. We see that, like in the neutral case, the bubbles exist for a finite region in parameter space, and in particular there is always a maximum radius β_{\max} for which the instability disappears.

⁹The analysis is consistent with the winding tachyon found to exist in [9], since the AdS soliton is the limit for which the dS_3 radius of curvature diverges, or by conformal transformation, the limit where $\beta \rightarrow 0$. For additional support for the conjecture in [8] see also [17].

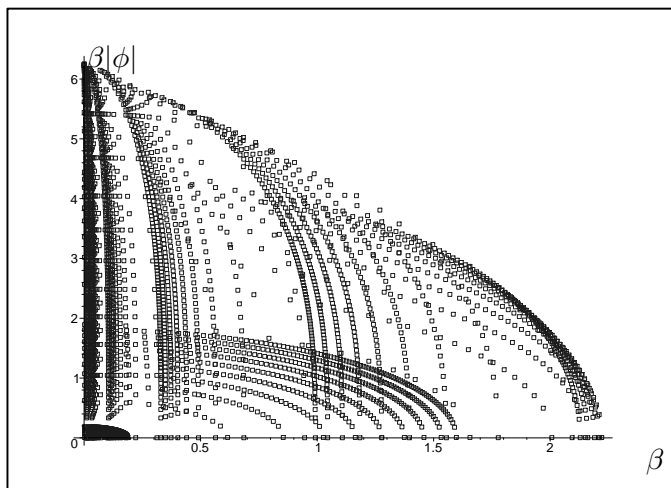


Figure 2: The existence of bubbles in the $(\beta, \beta|\phi|)$ plane in the one charge case.

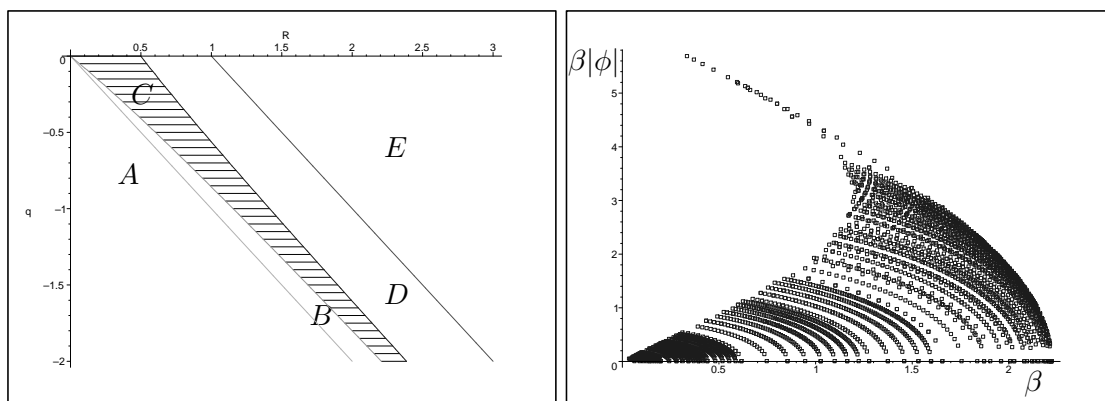


Figure 3: In the left diagram, instability can exist only in region C. No bubbles exist in region A, there is no negative mode in regions B and D, and region E corresponds to disappearance of the barrier $I_{\text{bub}} < 0$. In the right $(\beta, \beta|\phi|)$ diagram, the shaded region describes the region of possible instability.

Existence of the instability requires a negative mode,¹⁰ and in addition we need to satisfy the condition $I_{\text{bub}} > 0$ for metastability. We see in figure 3 that the region of instability always satisfies the condition for metastability, the transition is always first order.

Figure 3 shows the existence of instability, and the region of metastability, in parameter diagrams $(R = r_+^2, q)$ and $(\beta, \beta|\phi|)$. Interestingly we find both β and $\beta|\phi|$ are bounded. As a check we note that all the features of the neutral instability are reproduced when setting

¹⁰We note that due to the analytic continuation compared to the thermal interpretation, the existence of negative mode corresponds to a positive Hessian, instead of negative value.

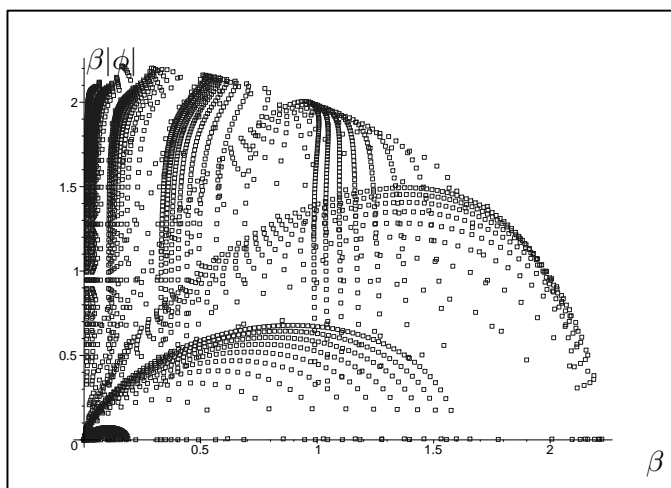


Figure 4: The existence of bubbles in the $(\beta, \beta|\phi)$ plane in the three equal charges case.

$\phi = 0$.

2.4 Three equal charges

As an additional example we now consider the case of three equal charges, $q_1 = q_2 = q_3 = q$, so that

$$f = 1 - \frac{\mu}{r^2} + r^2 H, H = \left(1 + \frac{q}{r^2}\right)^3 \tag{2.21}$$

In this case all the scalars X_i are constants, therefore the black hole is simply the AdS Reissner-Nordstrom solution, investigated for example in [16].

In this case we find that the physical parameters are

$$\begin{aligned} \beta &= \frac{2\pi R(R+q)^{3/2}}{2R^3 + R^2(1+3q) - q^3}, \\ \tilde{q}^2 &= q(R+q) \left(1 + \frac{1}{R}(R+q)^2\right), \\ \phi &= -\frac{\tilde{q}}{R+q}. \end{aligned} \tag{2.22}$$

where once again $q < 0$, corresponding to purely imaginary ϕ , for the bubble interpretation.

Similar to the discussion above, we first look at the region of parameters corresponding to the existence of a bubble spacetime. This is depicted in figure 4.

Finally, in figure 5 we show the regions for which the negative mode exists, and the region for which the topological black hole is meta-stable. As before we see that instability always occurs when the condition for metastability is satisfied, in other words the transition is always first order, and proceeds by tunneling over a potential barrier.

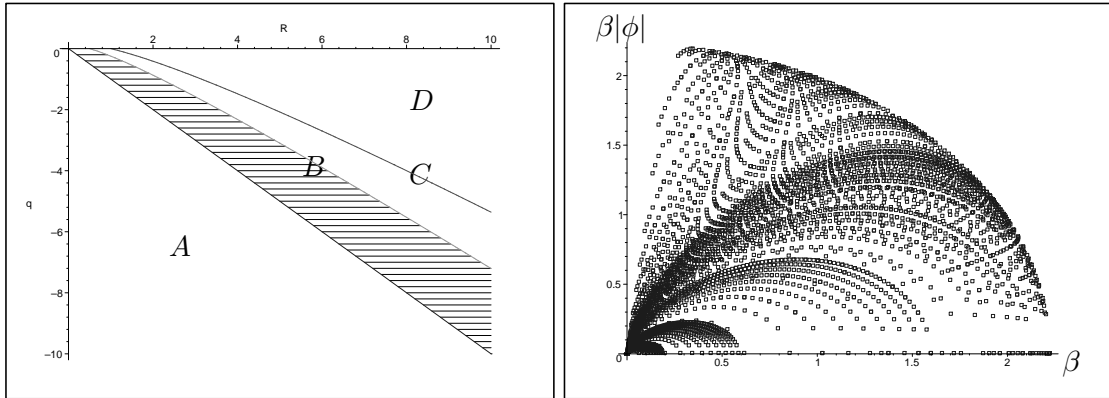


Figure 5: In the left diagram, instability can exist only in region B. No bubbles exist in region A, and region D corresponds to disappearance of the barrier $I_{\text{bub}} < 0$. In the right $(\beta, \beta|\phi)$ diagram, the shaded region describes the region of possible instability.

2.5 Features of the phase diagrams

Let us discuss qualitative features of the phase diagram, and how those are interpreted in terms of the quantum mechanics of a single variable (the winding condensate) and its effective potential. The winding mode (Wilson line) U is the lowest lying mode when compactifying on the circle, thus one can describe the long distance physics in terms of three dimensional scalar field theory living on dS_3 space.

We suggest that the physics of bubble nucleation can be effectively described in terms of the quantum mechanics of the spatially homogenous mode of U , integrating out the massive mode on the sphere S^2 in global coordinates. This approximation will manifestly break down at late times where the S^2 decompactifies, but may be sufficient for describing the bubble nucleation and early evolution.

Furthermore, since the bubble is nucleated at $t = 0$ when dS_3 is momentarily stationary,¹¹ we expect that for the process of the bubble nucleation the time dependence is unimportant,¹² though it may be important for a complete description of its subsequent evolution. As further evidence we note that the analysis in [18] of the asymptotic $R_t \times S^2 \times S^1$ boundary suggests that qualitative features to do with bubble nucleation are insensitive to the time dependence. Therefore we suggest to describe the process with the aid of quantum mechanics with an approximately time-independent Hamiltonian. That Hamiltonian can be calculated in the weak coupling limit [6], here we detail its features that can be read off from the bulk analysis for strong 'tHooft coupling.

One distinctive qualitative feature in the phase diagrams is the point where the Eu-

¹¹This is of course true only with particular choice of time, or Hamiltonian. In global coordinates the space is momentarily stationary at $t = 0$ which suggests that these coordinates are particularly convenient for describing the process of bubble nucleation, since reduction to quantum mechanics by integrating out the modes on S^2 is possible.

¹²For discussions of issues to do with time dependence in the bubble spacetime see for example [19, 20].

clidean action becomes small, which we would like to interpret as the disappearance in the barrier in the potential.¹³ Indeed, this happens for a small enough circle radius, and we find in all cases that near that point (where the circle parametrized by χ becomes string scale) a tachyon developed, which is described in the quantum mechanics of u , the winding condensate, as maximum in the effective potential. Note that the process of the winding tachyon condensation corresponds precisely to moving along the u axis. This indicates also that the true vacuum, even in the regimes where the decay is non-perturbative, is simply a winding mode condensate. The picture of the decay by the expanding bubble of nothing is therefore the conventional one, where a bubble of a true vacuum nucleates inside the false vacuum, and then exponentially expands outwards. The only novelty is that the true vacuum is a stringy non-geometrical phase.

The other striking feature of the phase diagrams is the disappearance of the instability. Indeed, in all cases there is a critical value of β , the circle's radius, for which the instability stops. Equivalently, since the boundary theory is conformally invariant, there is a minimal radius of curvature of the space dS_3 for which the instability stops. In particular, since the decay happens at the turning point $t = 0$ of dS_3 , there is a minimal size of the spatial sphere S^2 , and below that size the instability stops.

In conventional quantum mechanics of the homogeneous mode u , the natural explanation of that point is that the false and true vacua become degenerate in energy, and the instability stops. The bounce in that case disappears, while its limiting action stays finite. This is exactly the behavior we find when discussing the bounce near the critical value of β .

The quantum mechanics of the homogeneous mode is not sensitive to spatial dependence of the process. However, using the Euclidean black hole as an instanton leads to a puzzle related to its deSitter symmetry. This symmetry means that the decay is spatially homogeneous from the boundary viewpoint, for example the expectation value of the Wilson loop, $\langle U \rangle$ is homogeneous. This is contrary to the expectation one has from conventional bubble nucleation in first order transitions,¹⁴ where the typical situation involves an inhomogeneous pattern in space.

This pattern of decay is likely to be an artifact of the gravity limit. For bubble nucleation in finite space one has to compare the size of the space to the radius of a typical bubble. The latter is determined in terms of parameters of the potential, which are a function of the 'tHooft coupling λ . The suppression of the inhomogeneous decay is then an indication that typical bubble size is much larger than the size of the space, at least for large enough λ . In this case the decay will proceed at once throughout space.

In order to discover inhomogeneous decays in the gravity dual, we would have to construct solutions which break the deSitter symmetry (or the spherical symmetry in Euclidean space), and have lower action than the spherically symmetric case. However, from bulk considerations, such instantons are unlikely to dominate at the gravity limit. The dual field theory interpretation leads us to conjecture then that at small enough λ there ought to be a transition to non-spherically symmetric bubbles, which will then dominate the decay

¹³For a similar statement using a spacetime analysis [21] see [22].

¹⁴We thank Don Marolf for useful comments regarding this point.

process. Such transition is reminiscent of the Gregory-Laflamme instability, though in this case the localization is on a sphere.

3. Conclusions

We have studied the phase diagram of the maximally supersymmetric gauge theory on $dS_3 \times S^1$ with various configurations of gauge fields on the non-contractible circle. The results were interpreted in terms of quantum mechanics for a single variable (expectation value of the Wilson line around the circle). We have seen that various boundaries in those phase diagrams can be interpreted as features of the effective potential of that quantum mechanical system. This supports an interpretation of the decay mediated by the bubble of nothing as conventional vacuum decay, and the interpretation of the core of the bubble as the true vacuum, namely the tachyon condensate. This agrees with the interpretation of the tachyon condensation in the case of the AdS soliton, which is a limit of the bubble of nothing solutions we discuss.

Even though it is plausible that the dynamics effectively reduces to that of a single degree of freedom, we are not able to calculate the effective potential in the strong 'tHooft coupling regime. It is natural to attempt such calculation in other regimes of the gauge theory, and compare the qualitative features to those obtained here. A study of the same gauge theory at weak coupling is currently underway [6]. In addition to providing a definition of the effective potential, this study can shed some light on the mysterious "nothing" state, by viewing it from a conventional quantum mechanical perspective.

Additionally, it would be fascinating to probe the nothing state that apparently exists in the bulk bubble spacetime. It is likely that the picture in [23] applies here as well, and at the core of the bubble we have, in addition to the geometrical description, a winding mode condensate. String scattering in the background will then probe the winding condensate and will give indication that the geometrical description of the spacetime is incomplete. We hope to return to these issues in the near future.

Acknowledgments

We thank Vijay Balasubramanian, Micha Berkooz, Michael Dine, Gary Horowitz, Don Marolf, Rob Myers, Joan Simon, Evgeny Sorkin and Mark van Raamsdonk for useful and enjoyable conversations. This work is supported by discovery grant from NSERC of Canada.

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